Determination of the

Speed with which a Light Wave is Entrained

Crossing a Moving Medium

by

M. Hoek

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Already for a few years I had very much wished to know exactly the speed with which a light wave moves, when it is propagated in a preferred medium that is in a translational movement. In my studies the object has been the influence of the moving earth on the fundamental phenomena of optics, of which astronomy has proven useful (1), I had admitted with Fresnel that this speed is given by the formula ε $(1\text{-}1/n^2)$, where ε is the speed of the medium, and n is its absolute index of refraction. I then recognized that this relation was necessary not only to explain the celebrated experiment of Arago, which caused this formula to be introduced into science, but still to give an account of the circumstance why in astronomy one does not meet particular perturbations, dependant on the use of a beam-splitting prism.

Already Fizeau, by driving out a water column by the double tube of Arago, had shown that the relation mentioned was to be exact with a margin of 1/7. It was a first test to measure this quantity which appears intended to play a great part in the theory of optics, and whose exact knowledge is of a great interest for astronomy.

By modifying the experiment made by Fizeau, I succeeded in utilizing there the speed of revolution of the earth, which gives the advantage of simplifying the instrument and the occasion to determine with more precision the coefficient of entrainment.

Here how my apparatus was built. The source of light is an ordinary lamp, which lights the slit F (Pl.IX fig.5). The light coming from this slit, after having crossed the beam-splitting glass GG, is made parallel by the objective O. The rays which passed by the part E of this objective, meet on their way the tube TT filled with water and closed by glass, which I succeeded in placing exactly parallel. Then they enter the objective O_1, which makes them converge towards the point F_1. In this point, they meet towards F while following path F_1BAF. All the rays belonging to the beam under consideration thus cross again out of F.

(1) Researches astronomiques de l'observatoire d'Utrecht, livraison I.

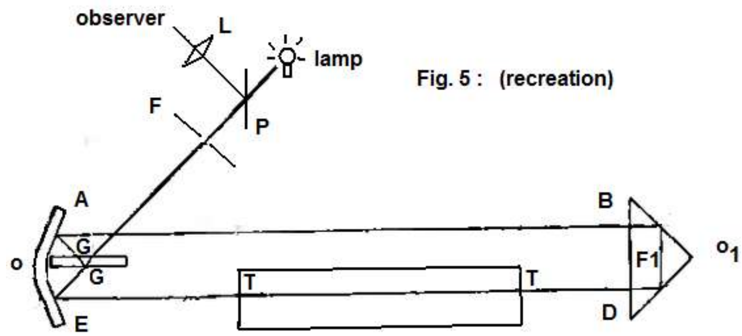

Fig. 5 : (recreation)

Another beam follows the opposite path. From A until past B it is propagated by the air, and it is only with the return that it meets tube TT. But before crossing out of F, all the light which traversed the apparatus meets the beam-splitter glass GG, which reflects a part of it towards f. This portion enters by the slit f the collimator C, it again is made parallel, analyzed by a prism P and is studied by means of the lens L.

It is obvious that in such an apparatus, as long as it is in rest, there is optical equivalence of the paths. But the phenomenon is more complicated as soon as apparatus begins a translating movement.

Let us admit that this movement has in direction A B, as the arrow in the figure indicates; each of the two beams is then continuously pulled by the mediums in which it is propagated. However one will have little trouble to recognize that, all being symmetrical in the apparatus, there is destruction of these effects in so far as they depend on the objectives. Indeed, the two rays test disturbances equal in parts A and E of the objective O, and of the same in the parts B and D of the O_1 objective. There is not gives any cause of delay there; it remains us, consequently, only to consider the influence of tube TT.

In this part of the apparatus, one of the beams is propagated in the direction of the moving earth, the other in a direction opposed to this movement. For one of them there is profit, for the other a loss. The equivalence of paths $FEDF_1BAF$ and $FABF_1DEF$ now gone, there should be a delay, and the spectrum must thus show black bands for any species of light whose half-length of wave is understood in this delay an odd number of times.

Here is my first experiment.

The experiment having been carried out, no band were shown. I initially studied my apparatus to make sure that there were no imperfections which hid the phenomenon. I modified it in several ways. I successively replaced of them all the various parts by others more perfect and corrected carefully, until using glasses of 1.3 and 2 meters focal distance. Lastly, I sought a combination which enabled me to reject the beam-splitting glass GG, waited until the light coming from only one point, after having crossed such a glass, seems to be part of several points belonging to an ellipsoid of revolution. Figure 6 represents such a combination.

The light coming from the slit F is made parallel by the objective a, and reflected partly towards b by an equilateral prism p q, whose face p is blackened. After having crossed in the point C, it is again made parallel by O, and traverses the remainder of the apparatus as in figure 5. On its return, after having passed the point C and the objective B, the light meets the prism. A part enters there by refraction, and, though weak, this portion gives enough of a perceptible spectrum to be examined by means of the lens L.

I will not need to add that the position of the instrument as well as the hours and dates of the experiments were selected so that the influence of the moving earth should have been felt.

Always the same result, no band was visible. This negative result having been found without a doubt for me, I dealt with the theoretical consequences, and I recognized that it completely confirms the coefficient of entrainment of Fresnel.

Let us admit that all the apparatus A B C has a movement of which speed either ε, and of which direction or B C, i.e. that of the arrow. Let us name the distances A B= L and B C = d, speeds of light λ in water, $n\lambda$ in the air. We will have:

1°. Actual speed of light ..... $= \lambda + \varphi$

Speed of tube .... $= \varepsilon$

Relative speed of light ...... $= \lambda + \varphi - \varepsilon$

time for light to traverse the tube .... $t_1 = \dfrac{L}{\lambda + \varphi - \varepsilon}$

2°. Speed of light in the air .... $= n\lambda$

Speed of mirror .... $= \varepsilon$

Relative speed .... $= n\lambda - \varepsilon$

time for the light to traverse distance BC .... $t_2 = \dfrac{d}{n\lambda - \varepsilon}$

3°. Speed of light in the air .... $= n\lambda$

Speed of point A .... $= \varepsilon$

Relative speed .... $= n\lambda + \varepsilon$

Time necessary to return to point A .... $t_3 = \dfrac{L + d}{n\lambda + \varepsilon}.$

Which gives:

$$t_1 + t_2 + t_3 = \frac{L}{\lambda + \varphi - \varepsilon} + \frac{d}{n\lambda - \varepsilon} + \frac{L + d}{n\lambda + \varepsilon} = T_1 \ldots\ldots (1)$$

In second bond, one can wonder what is the time necessary so that the light is propagated in the air of A and C, that it reconsiders its steps to meet have B the tube filled with water, then that it crosses this tube and reaches point A. By operating same manner, and by admitting the same translational movement, one will find:

$$t_4 + t_5 + t_6 = \frac{L + d}{n\lambda - \varepsilon} + \frac{d}{n\lambda - \varepsilon} + \frac{L}{\lambda - \varphi + \varepsilon} = T_2 \ldots\ldots (2)$$

So that there is no delay it is necessary to have only T1-T2 = 0, which makes it possible to calculate φ. We get:

$$L\left(\frac{1}{\lambda + (\varphi - \varepsilon)} - \frac{1}{\lambda - (\varphi - \varepsilon)}\right) + d\left(\frac{1}{n\lambda - \varepsilon} - \frac{1}{n\lambda + \varepsilon}\right) + \\ + (L + d)\left(\frac{1}{n\lambda + \varepsilon} - \frac{1}{n\lambda - \varepsilon}\right) = 0 \ldots (3)$$

or

$$L\left(\frac{2(\varphi - \varepsilon)}{\lambda^2 - (\varphi - \varepsilon)^2}\right) + L\left(\frac{2\varepsilon}{n^2\lambda^2 - \varepsilon^2}\right) = 0 \ldots\ldots (4)$$

or

$$(\varphi - \varepsilon)(n^2\lambda^2 - \varepsilon^2) + \varepsilon\,[\lambda^2 - (\varphi - \varepsilon)^2] = 0,$$

finally, by neglecting the quantities of the second order, i.e. ε^2 compared to $n^2\lambda^2$ and $(\varphi - -\varepsilon)^2$ per carry-forward with λ^2,

$$\varphi = \varepsilon\left(1 - \frac{1}{n^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The resulting negative result from this experiment provides a new demonstration of the known factor.

But there is more. One can say according to this experiment that this factor must be very-exact. To show this point one can reason in the following way. If φ had had zero value one would have found according to the formula (3) a delay

$$T_1 - T_2 = L\frac{2\varepsilon}{\lambda^2 - \varepsilon^2} - L\frac{2\varepsilon}{n^2\lambda^2 - \varepsilon^2}$$

or, by still neglecting the quantities of the second order,

$$T_1 - T_2 = \frac{2 L \varepsilon}{\lambda^2} \left(1 - \frac{1}{n^2}\right)$$

or finally, by expressing this delay in measure of length,

$$R = (T_1 - T_2)\, n\, \lambda = 2 L \frac{\varepsilon}{\lambda} \left(n - \frac{1}{n}\right).$$

In my experiment I had:

$$L = 100 \text{ m.m.} \quad n = 1\tfrac{1}{3} \quad \frac{\varepsilon}{\lambda} = \frac{1}{10000}$$

from where it follows R = 7/600 mm, or a spectrum with ten black bands. There was not even a delay of a half-length of wave of the line G, i.e. of 0.00022 mm. The experimental result is that the function:

$$\varphi = \varepsilon \left(1 - \frac{1}{n^2}\right)$$

is exact with a margin of 1/55.

It will have been noticed that the length of tube enters these last formulas. This provides us a means of determining our function φ with much more precision. I propose to repeat the experiment with a two meter long tube, which will lead to a determination 20 times more exact, one with the knowledge of the disturbances to which the function φ is prone.