Light Signals Sent Around a Closed Path

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IN the Sagnac experiment[1] two simultaneously emitted light signals are sent in opposite directions around a closed optical path, the whole optical system being in rotation about an axis perpendicular to the plane of the apparatus. The two signals upon returning to the point of origin on the apparatus are found to have taken different times, corresponding to velocities of light of $c+r\omega$ and $c-r\omega$, where c is the velocity of light as ordinarily measured by methods involving no rotation, r the radius of the circle described by the observation point and ω is the angular velocity of the light source and observation point as measured by a clock and scale on the supporting platform. The experiment was interpreted by its author as positive evidence for the existence of the luminiferous ether. It has been repeated recently by Dufour and Prunier[2] with the light source and observer separated from and not moving with the apparatus, with an identical result. In a discussion of this experiment Langevin[3] has asserted that by using "local time" on the apparatus the velocity of light is found to be "c." It has previously been dismissed by proponents of the theory of relativity as involving motion of rotation, and as such, along with the gyroscope, capable of explanation only by reference to the influence of all the matter in the universe, i.e., by attaching the pattern of radiant energy to a framework which is not called the ether.

It is the purpose of this paper first to show that the Sagnac experiment in its essentials involves no consideration of rotation, and second to investigate the results obtained when transported clocks are used. It will be shown that while it is true, if we use transported clocks, that the velocity of light measures "c," we must to obtain this result use a separate transported clock for each light beam, with the consequence that at each point of the apparatus we have two coincident clocks, each indicating a different time at the same instant.

For this study we shall consider a modified form of the experiment, the results of which are obviously to be expected to be identical with those obtained by Sagnac. We imagine (Fig. 1) a set of mirrors M_1, M_2, etc. in polygonal arrangement about an inscribed circle with center at 0. A light source S is arranged to move along a chord between two of the mirrors, and a light sensitive device associated with the light source records the emission of the light flash, and also its arrival after reflection around the series of mirrors. For the light signal proceeding in the clockwise direction (designated by the subscript 1) the position of the receiving device when it meets the returned signal is at S_1, for the counter clockwise signal its position is S_2. If t_1 and t_2 are the times of reception of the two signals in terms of absolute space and time (obtained by using measuring rods and clocks which are unaffected by the operations involved in their use) we have the relations:

$$\begin{aligned} P+vt_1&=ct_1,\\ P-vt_2&=ct_2, \end{aligned} \tag{1}$$

where P is the length of the path consisting of the series of chords. From these we have

$$\begin{aligned} t_1&=P/c-v,\\ t_2&=P/c+v. \end{aligned} \tag{2}$$

The difference in arrival time of the two light signals is

$$t_1-t_2=\frac{2Pv}{c^2(1-v^2/c^2)}. \tag{3}$$

Now we can increase the number of mirrors as much as we please, so that the length of the light path will approximate to any desired degree to the circumference of the inscribed circle, although, because of the rectilinear propagation of light, the light path is always a polygon. By moving the light source further from 0 along the radius the chords along which the light travels

[1] Comptes rendus 157, 708 and 1410 (1913).
[2] Dufour and Prunier, Comptes rendus 204, 1925 (1937).
[3] Langevin, Comptes rendus 205, 304 (1937).

approximate to arcs of the circle. We can therefore obviously substitute circular coordinates for the rectangular coordinates thus far used. Doing this relations (2) and (3) become

$$t_1=\frac{2\pi r}{c-r\omega},\qquad t_2=\frac{2\pi r}{c+r\omega} \tag{4}$$

and

$$t_1-t_2=\frac{4\pi r^2\omega}{c^2-r^2\omega^2}=\frac{4\pi(r^2\omega/c^2)}{1-(r^2\omega^2/c^2)}. \tag{5}$$

This last formula agrees with the result of the Sagnac experiment (in which the accuracy attained is only sufficient to show the first order in $r\omega/c$), where the whole apparatus is rotated, and shows, what is indeed obvious by a detailed study of the light paths, that the changes of direction of the several mirrors in the apparatus, produced by the rotation, have a negligible effect on the *length* of the light path. The essential feature is that the light signals are sent around a closed circuit, with motion meanwhile of the receiving device. As a limiting case, let the mirrors coalesce into a circular shell. It is clear that it is then indifferent whether the shell rotates about the center 0, or is stationary. This leaves us with the essential condition that the receiving element shall move with respect to the pattern of radiant energy with which the light signal, which is independent of the velocity of the source, and of the (rotational) velocity of the mirror system, becomes a part.

We now turn to the measurements of arrival time of the light signals which would be made by clocks which suffer a change of rate according to the relation

$$\nu=\nu_0(1-v^2/c^2)^{\frac{1}{2}}.$$

We first consider the modifications of formulae (2), (3) and (4) which occur if the angular velocity of the system is measured by a clock attached to and moving with the light source. Since the time measures less, by the factor $(1-v^2/c^2)^{\frac{1}{2}}$, we have the modified formulae

$$t_1'=\frac{P(1-v^2/c^2)^{\frac{1}{2}}}{c-v},\qquad t_2'=\frac{P(1-v^2/c^2)^{\frac{1}{2}}}{c+v} \tag{2a}$$

and

$$t_1'-t_2'=\frac{2Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}. \tag{3a}$$

We now proceed to study the values to be obtained if the time of arrival of the light signals is measured by clocks which are transported to the point of reception of the signals, that is by "local time." In order to use the results of previous papers[4] we shall transport these clocks in a carefully specified way, deviations from which in the actual Sagnac experiment can be evaluated as to their significance later. This special procedure consists in carrying the light source and receiving device on a moving band, which travels along the chords between the mirrors[5] with the velocity v. Divisions on this band are contracted by motion in the ratio $(1-v^2/c^2)^{\frac{1}{2}}$: 1. The clocks to be used to measure

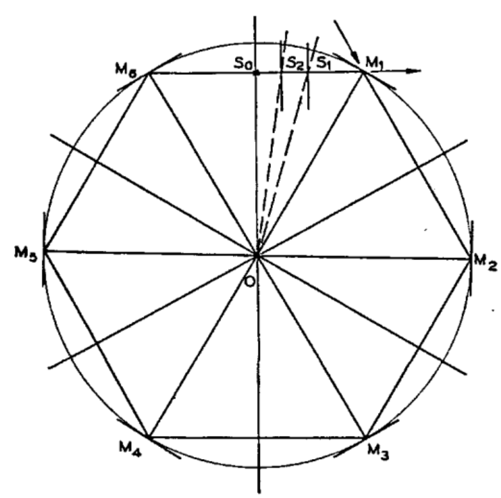

FIG. 1.

[4] J. Opt. Soc. Am. 27, 263 (1937).

[5] In order to avoid any question of what happens to the clock rates or scale readings on the moving band when it changes direction at each mirror, we can substitute for the single moving band a series of bands moving without change of direction whose paths cross at M_1, M_2, etc. (as suggested by the arrow at M_1 in the figure), at which points the clock and scale readings are transferred from one band to the other.

the arrival times of the light signals at the mirrors and on their return to the source, are transported along this moving band, at some small velocity W, in the direction of travel of the two light signals, that is we have two clocks, of velocities of transport $+$ and $-W$.

We then have, following the procedure of a previous paper,[4] that clocks so transported, compared with a clock at the origin, are set back by

$$\Delta S=\frac{P}{W}[(1-v^2/c^2)^{\frac{1}{2}}-(1-(v\pm W)^2/c^2)^{\frac{1}{2}}]. \quad (6)$$

Solving this by the methods previously used[4] we get for the altered settings of the clocks

$$\Delta S_1=-\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}$$

for a clock transported in the clockwise direction,

(7)

$$\Delta S_2=+\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}$$

for a clock transported in the counterclockwise direction.

The time read by such a clock will be the true time, reduced by the factor $(1-v^2/c^2)^{\frac{1}{2}}$, plus the change in setting. For the time so read for the signal traveling clockwise, by the clock transported clockwise, we have

$$t_R=\frac{P}{c-v}(1-v^2/c^2)^{\frac{1}{2}}-\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}} = \frac{P}{c(1-v^2/c^2)^{\frac{1}{2}}} \quad (8)$$

and for the signal traveling counterclockwise, by the clock transported counterclockwise

$$t_R=\frac{P}{c+v}(1-v^2/c^2)^{\frac{1}{2}}+\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}} = \frac{P}{c(1-v^2/c^2)^{\frac{1}{2}}}. \quad (9)$$

We also have that the path P, in terms of the contracted divisions on the moving band measures as

$$P_R=\frac{P}{(1-v^2/c^2)^{\frac{1}{2}}}. \quad (10)$$

We thus have finally, for the measured velocity of either signal

$$\frac{P_R}{t_R}=\frac{\dfrac{P}{(1-v^2/c^2)^{\frac{1}{2}}}}{\dfrac{P}{c(1-v^2/c^2)^{\frac{1}{2}}}}=c \quad (11)$$

which is the result favored by Langevin.

Now in order to find where we stand, having obtained this result, by the use of "local time," let us look at the two clocks which have been brought back into coincidence at the light source, each having traveled around the optical path in the direction of one of the light signals. When we do this we find *each indicating a different time.* Their difference of indication is

$$\Delta S_2-\Delta S_1=\frac{2Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}. \quad (12)$$

Looking back at the expression for the difference of arrival times of the light signals, as recorded by a clock traveling with the light source, namely (3a), we find it is *identical* with that just obtained. All we have done by using "local time" is to substitute for the observation of two different arrival times on one clock, the observation of identical arrival times on two clocks, which differ in their settings by just the difference in arrival times shown by the single clock. The performer of the experiment is thus left with the alternative of accepting the observed arrival of the signals at different times as a fact, or, if he must cling to a constant value for the measured velocity of light, of putting his faith in carefully labeled clocks which tell him the signals arrive at the same time. But he must avoid looking at both[6] clocks at once! In short

[6] There are of course not merely two clocks, but an infinity of clocks, when we include those which could be transported at finite speeds, and around other paths. As emphasized previously the idea of "local time" is untenable, what we have are *clock readings*. Any number of clock readings at the same place are physically possible, depending on the behavior and history of the clocks used. More than one "time" at one place is a physical absurdity.

the physical fact cannot be evaded by juggling the measuring instruments.

In conclusion the relationship of the experiment here discussed, with its stationary polygonal arrangement of mirrors, and its moving band, on which the measurements are made, to the actual Sagnac experiment may be briefly considered. Let us reduce the Sagnac experiment to the rotating cylindrical mirror shell above suggested, and let the light source be carried on a chord of that shell. In order to go over to this case we need to know what contraction of dimensions, if any, the shell experiences on rotation, and what change of rate a clock experiences when transported over a curved path. Without knowing these we can nevertheless from general considerations predict that the phenomena will be altered from those we have studied only by second-order times in v/c or $r\omega/c$. If our path length is measured by divisions on the ring these divisions will be contracted exactly as the ring, so that the measured path length will always be P, instead of $P/(1-r^2\omega^2/c^2)^{\frac{1}{2}}$ as obtained from the moving band. With regard to the true times of travel of the light signals, these will be directly affected in proportion to the contraction of the ring, which from consideration familiar in connection with the Ehrenfest paradox must be less than $(1-r^2\omega^2/c^2)^{\frac{1}{2}}$. The alteration of clock rate by transport in a curved path can only differ from $(1-r^2\omega^2/c^2)^{\frac{1}{2}}$ by small terms involving $1/r$ since as r is increased it must approximate to $(1-r^2\omega^2/c^2)^{\frac{1}{2}}$ as a limit. We can therefore always insure, by making our apparatus large enough, that the latter factor is substantially correct. It is therefore apparent that our results must hold at least as far as the first order of $r\omega/c$ for the actual Sagnac experiment.

The net result of this study appears to be to leave the argument of Sagnac as to the significance of his experiment as strong as it ever was. The suggested use of "local time" merely offers another way of measuring the effect of rotating the apparatus, namely in terms of the differences between two clocks carried around a circuit, instead of differences of arrival time of two light signals sent around the same circuit. The rotation, which can be measured in either of these ways, is not relative rotation of the apparatus with respect to the platform on which it is mounted, or to the laboratory—either of these might be rotated with respect to the apparatus, with no resultant Sagnac effect. The observer on the apparatus has just one reference framework by which he can predict whether the Sagnac effect will appear or not; that framework is the pattern of radiant energy from the stars. If his apparatus rotates with respect to the stars he will observe a Sagnac effect, if it does not, then no matter how great relative rotation it exhibits with respect to its material surroundings, there will be no Sagnac effect.