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The Aberration of Clocks and the Clock Paradox

HERBERT E. IVES
Bell Telephone Laboratories, New York, N. Y.
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THE ABERRATION OF CLOCKS

In a preceding paper,[1] I have shown that clocks experiencing the Larmor-Lorentz frequency change are altered in setting by transport by exactly the amount necessary to make the measured times of transit of light signals invariant on a uniformly moving body, subject to the Fitzgerald contraction. Study of this change of setting suggests that if clocks are transported over the same measured distance at different times, when the velocity of the observing platform is different, their settings should differ.

From the preceding paper, I take the expression for the change in setting of a clock moved between two points at the distance L_s apart (when the observing platform is stationary) at the observed velocity W_0, the velocity of the observing platform through the ether being v. It is

$$\Delta S = \frac{(L_s/c)(1-v^2/c^2)^{\frac{1}{2}}}{\dfrac{(W_0/c)(1-v^2/c^2)}{W_0 v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}}\left[(1-v^2/c^2)^{\frac{1}{2}}-\left\{1-\frac{\left(v+\dfrac{W_0(1-v^2/c^2)}{W_0v/c^2+(1+W_0^2/c^2)^{\frac{1}{2}}}\right)^2}{c^2}\right\}^{\frac{1}{2}}\right]. \tag{1}$$

To solve this, put

$$v/c = x, \tag{2}$$

$$W_0/c = \alpha, \tag{3}$$

$$z = \frac{\alpha(1-x^2)}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}. \tag{4}$$

Eq. (1) then becomes

$$\Delta S = \frac{(L_s/c)(1-x^2)^{\frac{1}{2}}}{z}[(1-x^2)^{\frac{1}{2}}-(1-(x+z)^2)^{\frac{1}{2}}]; \tag{5}$$

then

$$x+z = \frac{\alpha+x(1+\alpha^2)^{\frac{1}{2}}}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}},$$

$$1-(x+z)^2 = \frac{1-x^2}{[x\alpha+(1+\alpha^2)^{\frac{1}{2}}]^2} \tag{6}$$

[1] "Light Signals on Moving Bodies as Measured by Transported Rods and Clocks," J. O. S. A. **27**, 263 (1937).

and

$$(1-(x+z)^2)^{\frac{1}{2}}=\frac{(1-x^2)^{\frac{1}{2}}}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}},$$

so that

$$\begin{aligned}\Delta S&=\frac{L_s}{c}\frac{(1-x^2)}{\alpha-(1x^2)}[(x\alpha+(1+\alpha^2)^{\frac{1}{2}}]\left(1-\frac{1}{x\alpha+(1+\alpha^2)^{\frac{1}{2}}}\right)\\ &=\frac{L_s}{c\alpha}(x\alpha+(1+\alpha^2)^{\frac{1}{2}}-1)\\ &=\frac{L_s}{c}\left[x-\frac{1-(1+\alpha^2)^{\frac{1}{2}}}{\alpha}\right] \qquad (7)\\ &=\frac{L_s}{c}\left[\frac{v}{c}-\frac{1-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right].\end{aligned}$$

Suppose that the observing platform is moving with the velocity v_1 at one time, and later is moving with the velocity v_2. Let us at some time during the first period move a clock between two fixed points on the platform, whose distance apart, for the platform stationary, is L_s. If we measure this distance by rods placed successively end to end (i.e., moving infinitely slowly with respect to the platform) we simply measure the distance as L_s, its true value being $L_s(1-v_1^2/c^2)^{\frac{1}{2}}$. According to (7) the setting of the moved clock is changed by

$$\Delta S_1=\frac{L_s}{c}\left[\frac{v_1}{c}-\frac{1-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]. \qquad (8)$$

Let us now, at some time during the second period, move another clock (which may be the clock left at the origin) to the distant point. Its change of setting, by (7) is

$$\Delta S_2=\frac{L_s}{c}\left[\frac{v_2}{c}-\frac{1-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]. \qquad (9)$$

The two transported clocks will then differ in setting by

$$\Delta S_2-\Delta S_1=\frac{L_s}{c}\left[\frac{v_2-v_1}{c}\right]. \qquad (10)$$

If we put T for the time taken by a light signal to traverse the path when the platform is stationary we have

$$\Delta S_2-\Delta S_1=T\left[\frac{v_2-v_1}{c}\right]. \qquad (11)$$

Let us now compare this operation with the familiar phenomenon of the aberration of light, as disclosed by observations on a star made with a telescope. In that case we have that the deviation of the star image between epochs is

$$\Delta\sigma=F\left[\frac{v_2-v_1}{c}\right], \qquad (12)$$

where F is the focal length of the telescope. We therefore have expressions of similar form for the different positions of a stellar image and the different settings of transported clocks, the one in terms of the *distance* traversed by a light signal, the other in terms of the *time* of transit of a light signal, both measures of the change of velocity of the observing platform with respect to the ether.

It is assumed in the derivation of (11) that the rate of the clock originally transported continuously follows the changing velocity of the platform, without experiencing a change of setting. The factor which gives the clock rate, $(1-v^2/c^2)^{\frac{1}{2}}$, is thus imagined as describing an effect of the nature of a frictional force holding at all instants. In any case the change of setting of clocks at the origin and at the distant point in the experiment as outlined would be the same, since both experience the same change of velocity.

If the velocity of transport of the clocks is made infinitesimal, the change of clock setting approximates the change of transit time of light signals, due to the alteration in the velocity of the platform. In this case the aberration of clocks would exhibit itself as a discrepancy between the times indicated by a continuously running clock at a distant point, and time settings derived from light signals assigned the velocity c.

The clock phenomenon above discussed would be complicated, in the case of the earth, by the diurnal rotation, which would be periodically adding to and subtracting from the velocities of the clocks due to the earth's orbital motion. In order to meet the condition that clocks at both ends of the path should experience the same changes of velocity, the experiment would have to be performed at the same *sidereal* time, at different epochs. Practically, the effect would be beyond the possibility of detection by any clocks now available. The difference of setting due to the reversal of direction of the earth's orbital motion at epochs six months apart is a fraction of the order of 10^{-4} of the time required for a light signal to traverse the path on the earth, which would in representative experimental conditions be of the order of 10^{-4} second. This would require the originally transported clock to maintain its time for six months to better than 10^{-8} second.

As stated above, this aberration of clocks gives a measure of the change of velocity of the earth with respect to the ether, similar to the aberration of light exhibited by the displacement of a star image in the telescope. It differs from the aberration of light in that it could, theoretically, be detected by observations confined solely to the earth, provided the path of clock transportation were constant in direction, as fixed by a Foucault pendulum or gyroscope. It does not require an energy pattern such as that set up by the stars and observed in a telescope in order to render the shift of the ether with respect to the earth detectable. On the other hand, it does not give the overwhelming probability that it is the earth rather than the ether which moves annually, which is furnished by stellar aberration due to the vast number of stars, as discussed in the previous paper.

The Clock Paradox

In the preceding paper[1] it was shown that the expressions describing the transit of a light signal simultaneously over several uniformly moving bodies could be formulated entirely in terms of the observed relative velocities of the bodies (and the velocity of light). From this result it is tempting to conclude that only relative velocities of material bodies are significant (or existent), and that the occurrence of similar relative motions must always be accompanied by completely similar phenomena.

A hypothetical experiment based on this conclusion leads to the "clock paradox." Given two similar clocks A and B; let B be moved to a distant point at the velocity V with respect to A, and back to A. It will, because of its motion, go at a slower rate, a function of V^2/C^2, and will on its return be *slow with respect to A*. But if relative motion only is of significance, A is likewise moving at the velocity V with respect to B, and hence will, when the clocks are together again, *be slow with respect to B*. Obviously two clocks, side by side, cannot each be slow with respect to the other. There is here a logical inconsistency which demands an examination of the premises.

I propose to discuss this problem from the standpoint of the preceding paper, in which the ether is assumed as the framework with respect to which the alteration of clock rates takes place. It will be shown that, so studied, the behavior of clocks is entirely rational; the "clock paradox" does not arise. As in the case of light signals, velocities with respect to the ether do not appear in the final formulae; but a given relative velocity of two clocks, separated and united, is entirely compatible with one changing its setting, and the other not. In other words, the clock phenomena, unlike the light signal phenomena discussed, are *not* unique functions of the observed relative velocities.

Let us for simplicity first consider that one of our clocks, A, is moving uniformly through the ether, and that the other clock, B, is moved away from A, and back again at a uniform observed velocity. We shall, as before, look upon the change of clock rate, due to motion, as similar to a frictional effect; the operation of reversing the direction of motion introducing of itself no change of setting. Our problem is to find the total change of setting of B due to its motion out and back.

The appropriate expressions, of the form given by Eq. (4) of the previous paper, are

$$\Delta S=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{W_1}\times[(1-v^2/c^2)^{\frac{1}{2}}-(1-(v+W_1)^2/c^2)^{\frac{1}{2}}] \quad (13)$$

and

$$\Delta S'=\frac{L_s(1-v^2/c^2)^{\frac{1}{2}}}{W_2}\times[(1-v^2/c^2)^{\frac{1}{2}}-(1-(v-W_2)^2/c^2)^{\frac{1}{2}}], \quad (14)$$

where W_1 and W_2 are the velocities of clock B

with respect to A on its journeys out and back. The solution of (13) in terms of an observed velocity W_0 has already been obtained in (7). To solve (14) we derive, by exactly similar processes to those already given,

$$W_2=\frac{W_0(1-v^2/c^2)}{(1+W_0^2/c^2)^{\frac{1}{2}}-W_0v/c^2} \qquad (15)$$

and finally

$$\Delta S'=\frac{L_s}{c}\left[\frac{v}{c}-\frac{1-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]. \qquad (16)$$

The sum of ΔS [from (7)] and $\Delta S'$ [from (14)], is the amount by which B is set back with respect to A, by its journey; it is

$$\Delta S+\Delta S'=-\frac{2L_s}{c}\left[\frac{1-(1+W_0^2/c^2)^{\frac{1}{2}}}{W_0/c}\right]. \qquad (17)$$

(For small values of W_0/c, this reduces to $\frac{1}{2}T(W_0^2/c^2)$, where $T=2L_s/W_0$.) We learn from this equation that the change of setting of B is independent of v, that is, it is the same whether A is moving uniformly or is stationary with respect to the ether, and that the change of setting of B is expressible in terms of the observed relative velocity of the two clocks.

Let us now take the case where clock B is moving uniformly through the ether, while clock A is moved out and back in the opposite direction to the previous excursion of B. Inspection of the Eqs. (13) to (17) shows that for this second case we obtain *an identical expression* (17) for the amount by which A is set back with respect to B. As a third case consider *both* clocks moved out and back with respect to their uniformly moving locus of coincidence, at velocities such that their relative measured velocity is again W_0. It is clearly possible for their velocities to be such that they each experience the *same* change of setting, with the result that there is *no difference of setting* between the clocks.

It is clear from consideration of these special cases that it is not possible for observers on A and B to predict from their observations of relative velocity in what relation their clocks will stand on returning to coincidence. There exists a series of possible setting differences all compatible with the same observed relative velocities. Eq. (17) gives the *maximum* change of setting which may occur for an observed relative velocity W_0; the actual difference, not predictable from the observed relative velocities alone, may vary between this maximum and zero. When a difference of setting actually occurs that clock of two which is fast, after their separation and reunion, is the one which has moved least with respect to the motion of their point of coincidence through the ether.

In the actual performance of an experiment on these changes of clock settings a common material platform would, for practical reasons, be provided, with reference to which the two clocks would be moved. Our point of intersection of clock motions would then be a point on this platform. What our study has shown is that the differences in clock indications, after the clocks have been moved in various ways with respect to this reference point, are given by Eq. (17) and the similar equations called for by the discussion immediately following. In these equations the state of rest or uniform motion of the reference platform drops out, so that the results are functions only of the relative motions of the clocks *with reference to the uniformly moving platform.* In this sense only can we say that relative motions of matter are alone significant in this problem. The idea that the relative motions of the clocks, *with reference to each other*, are sufficient to predict the resultant conditions, is fallacious. The clock paradox originates in this fallacious conclusion.

The behavior of separated and reunited clocks is very clearly comprehended when we correlate it with the aberration of light. Suppose that observers on each clock observe the variation of position of the stars in the sky during the interval of relative motion of the clocks. If no stellar aberration is observed from one clock, that is the clock whose setting will be unchanged. If the stars are seen from one clock to exhibit a to and fro motion, then that clock will be found to be set back when the clocks are again side by side. If aberration is observed from both clocks, their final difference of setting will be a function of the difference in the observed oscillations of stellar position. In short, in order to predict what changes in setting will occur in clocks moved apart and then together, we must observe their motions with respect to the ether (locus of stellar light patterns)[2] we cannot determine these changes by observations on their motion with respect to each other.

We thus find that the study of moved clocks, using the Bradley-Fresnel ether, together with the Fitzgerald-Larmor-Lorentz contractions of length and frequency, indicates an entirely consistent and reasonable behavior. The "clock paradox" is a consequence of a sweeping and unqualified application of the hypothesis that relative motion of matter is the only operative factor. This hypothesis is shown by the above study not to be a universal consequence of the length and frequency contractions postulated to explain the Michelson-Morley experiment. That experiment requires processes to account for invariance with respect to uniform motion through the ether; these are furnished by the contractions. There are no relative motions of matter in the Michelson-Morley experiment, and no hypothesis with regard to relative motion of matter is required or involved. Our study shows that, on the basis of the length and frequency contractions, the moved clocks have the same property as the Michelson-Morley apparatus, namely invariance of behavior with respect to the state of rest or motion of *the whole system of clocks* through the ether. The supplementary hypothesis, suggested by extrapolation from the behavior of light signals, that relative motions are alone significant in all cases, is discredited by the physical and logical absurdity of the "clock paradox." The use of the ether as the reference frame is supported by its ability to give a rational account of clock behavior.[3]

[2] The state of rest or uniform motion of the material platform of the preceding paragraph must in the last analysis be determined by recourse to observations on aberration, so that the findings of that paragraph are included in this conclusion.

[3] Another case where confusion results from a loose application of the idea that relative motion of matter is alone significant, is the problem of the aberration of light from binary stars. Spectroscopic binaries, whose components have orbital velocities of the order of magnitude of the earth's, would, if the relative velocities of earth and star were the sole determining factor, necessarily be telescopic binaries with separations of the order of the aberration constant (20″). Such spectroscopic binaries exist, which are not telescopically resolvable. When aberration is ascribed to the *observer's* motion through the ether no such difficulty arises.